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Book Reviews

Communicated by P. Hajnal

Carles Casacuberta, Rosa Maria Miró-Roig, Joan Verdera and Sebastiá Xambó-Descamps(Eds.), **European Congress of Mathematics** (Vol. I and II, Proceedings of the 3rd Congress (3ecm) held in Barcelona, July 10–14, 2000, Progress in Mathematics, 201 and 202), l+582 pages and xii+641 pages, Birkhäuser Verlag, Basel, 2001.

The forward of this two volumes describes the facts and motivations of this congress: "The Third Europen Congress of Mathematics (3ecm) took place from July 10th to July 14th, 2000 in Barcelona. It was organised by the Societat Catalana de Mathemàtiques (Catalan Mathematical Society), under the auspices of the European Mathematical Society (EMS). As foreseen by the EMS and the International Mathematical Union, this Congress was a major event in World Mathematical Year 2000. In addition to reviewing outstanding research achievements, important aspects of the life of European mathematics were discussed."

"Mathematics is undergoing a period of rapid changes. Effective computation and applications in science and technology go over more hand in hand with conceptual developments. It was one of the aims of 3ecm to reflect this mutual enrichment, while steering present and future trends of mathematical sciences. In fact the motto of the Congress, *Shaping the 21st Century*, was meant to capture these views."

"Nearly 1400 people from 87 coutries garhered together in the Palau de Congressos of Barcelona in order to take part in the activities of the 3ecm scietific programme: Nine plenary lectures, thirty invited lectures in parallel sessions, lectures given by EMS prize winners, ten mini-simposia on special topics, seven round tables, poster sessions, presentations of mathematical software and video exhibitions. Twenty events were satellites of 3ecm in various countries."

Finally we give the list of papers presented in this important document of the mathematical community:

Talks by Plenary Speakers: Robert Dijikgraaf, *The Mathematics of M-Theory*; Hans Föllmer, *Probabilistic Aspects of Financial Risk*; Jendrik W. Lenstra, Jr., *Flags and Lattice Basis Reduction*; Yuri I. Manin, *Moduli, Motives, Mirrors*; Yves Meyer, *The Role of Oscillations in Some Nonlinear Problems*; Carles Simó, *New Families of Solutions in N-body Problems*; Marie-France Vignéras, *Irreducible Modular Representations of Reductive p-Adic Group and Simple Modules for Hecke Algebras*; Oleg Viro, *Dequantization*

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of Real Algebraic Geometry on Logarithmic Paper. **Articles by Parallel Speakers:** Rudolf Ahlswede, *Advances on Extremal Problems in Number Theory and Combinatorics*; Volker Bach, *Spectral Analysis of Nonrelativistic Quantum Electrodynamics*; Viviane Baladi, *Spectrum and Statistical Properties of Chaotic Dynamics*; Joaquim Bruna, *Sampling in Complex and Harmonic Analysis*; Nicolas Burq, *Lower Bounds for Shape Resonance Widths of Schrödinger Operators*; Xavier Carabré, *A Conjecture of De Giorgi on Symmetry for Elliptic Equations in* $\mathbf{R}^n$; Peter J. Cameron, *The Random Graph Revisited*; Zoé Chatzidakis, *Difference Fields: Model Theory and Applications to Number Theory*; Ciro Ciliberto, *Geometric Aspects of Polynomial Interpolation in More Variables and of Waring's Problem*; Gianni Dal Maso, *The Calibration Method for Free Discontinuity Problems*; Jan Denef and Francois Loeser, *Geometry on Arc Spaces of Algebraic Varieties*; Barbara Fantechni, *Stacks for Everybody*; Alexander B. Goncharov, *Multiple* ζ*-Values, Galois Groups, and Geometry of Modular Varietis*; Alexander Gigor'yan, *Heat Kernels on Mainfolds, Graphs and Fractals*; Michael Harris, *Local Langlands Correspondences and Vanishing Cycles on Shimura Varieties*; Renato Ituriaga and Konstantin Khanin, *Burgers Tubulence and Dynamical Systems*; Kurt Johansson, *Random Growth and Random Matrices*; Pekka Koiskela, *Sobolev Spaces and Quasiconformal Mappings on Metric Spaces*; Nicholas S. Manton and Bernard M.A.G. Piette, *Understanding Skyrmions Using Rational Maps*; Ieke Moerdijk, *Models for the Leaf Spce of a Faliation*; Eric M. Opdam, *Multivariable Hypergeometric Functions*; Thomas Peternell, *Contact Structures, Rational Curves and Mori Theory*; Alexander Reznikov, *Analytic Topology*; Bernhard Scmidt, *Towards Syser's Conjecture*; Klaus Schmidt, *The Dynamics of Algebraic* $\mathbf{Z}^d$*-Actions*; Bálint Tóth, *Self-Interacting Random Motions*; Erik van den Ban and Henrik Schlichtkrull, *Harmonic Analysis on Reductive Symmetric Spaces.* **Articles by Prize Winners:** Semyon Alesker, *Classification Results on Valuations on Convex Sets*; Raphaël Cerf, *Towards a Microscopic Theory of Phase Coexistence*; Dominic Joyce, *Constructing Compact 8-Manifolds with Holonomy Spin*(7) *from Calabi-Yau Orbifolds*; Vincent Lafforgue, *Banach KK-Theory and the Baum-Connes Conjecture*; Michael McQuillan, *An Introduction to Non-Commutative Mori Theory*; Stefan Nemirovski, *Geometric Methods in Complex Analysis*; Paul Seidel, *Vanishing Cycles and Mulation*; Wendelin Werner, *Critical Exponents, Conformal Invariance and Planar Brownian Motion.* **Mini-Symposium on Computer Algebra:** Manuel Bronstein, *Computer Algebra Algorithms for Linear Ordinary Differential and Difference Equations*; Wolfram Decker, *Some Introductory Remarks on Computer Algebra*; Gaston H. Gonnet, *A Study of Iteration Formulas for Root Finding, Where Mathematics, Computer Algebra and Software Engineering Meet*; Laureano González-Vega and Tomás Recio, *Industrial Applications of Computer Algebra: Climbing Up a Moutain, Going Down a Hill*; Gert-Martin Greuel, *Applications of Computer Algebra to Algebraic Geometry, Singularity Theory and Symbolic-Numerical Solving.* **Mini-Symposium on Curves over Finite Fields and Codes:** Noam D. Elkies, *Explicit Towers of Drinfeld Modular Curves*; Arnaldo Garcia, *Curves over Finite Fields Attaining the Hasse-Weil Upper Bound*; Farshid Hajir and Christian Maire, *Asymptotically Good Towers of Global Fields with Many Rational Places*; Gerard van der Geer, *Curves over Finite Fields and Codes*; Chaoping Xing, *Authentication Codes and Algebraic Curves.* **Mini-Symposium**

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on Free Boundary Problems: Giovanni Bellettini, *Some Aspects of Mean Curvature Flow in Presence of Nonsmooth Anisotropies*; Klaus Deckelnick, *A Phase-Field Model for Diffusion-Introduced Grain Boundary Motion*; Irina V. Denisova, *Evolution of a Closed interface between Two Liquids of Different Types*; Gonyalo Galiano, *Applications of a Local Energy Method to Systems of PDE's Involving Free Boundaries*; Harald Garcke, *Phase Boundaries in Alloys with Elastic Misfit*; Josephus Hulshof, *Some Apects of the Thin Film Equation*; Regis Monneau, *A Brief Overview on the Obstacle Problem*; Henrik Shahgholian, *The Impact of Monotonicity Formulas in Regularity of Free Boundaries*; José Miguel Urbano, *A Free Boundary Problem: Contributions from Modern Analysis.* **Mini-Symposium on Mathematical Finance: Theory and Practice:** Tomasz R. Beielcki and Stanley R. Pliska, *Risk Sensitive Control with Applications of Fixed Income Portfolio Management*; M.A.H. Dempster and A. Eswaran, *Wavelet Based PDE Valuation of Derivatives*; Ernst Eberlein and Sebastian Raible, *Some Analytic Facts on the Generalized Hyperbolic Model*; Hélyette Geman, *Functionals of Brownian Motion in Path-Dependent Option Valuation*; Ton Vorst, *Optimal Portfolios under a Value at Risk Constraint.* **Mini-Symposium on Quantum Chaology:** Eugene Bogomolny, *Trace Formulas and Spectral Statistics of Diffactive Systems*; Monique Combescure and Didier Robert, *Semiclassical Result in the Linear Response Theory*; Jens Marklof, *The Berry-Tabor Conjecture*; Zeév Rudnick, *On Quantum Unique Ergodicity for Linear Maps of the Torus.* **Mini-Symposoium on Quantum computing:** Nicolas Gisin, Renato Renner and Stefan Wolf, *Bound Information: The Classical Analog to Bound Quantum Entanglement.* **Mini-Symposium on String Theory and M-Theory:** Nischael R. Douglas, *D-Branes on Calabi-Yau Manifolds*; José M. F. Labstida, *Knot Invariants and Chern-Simons Theory*; Marcos Mariño, *Topological Quantum Field Theory and Four-Manifolds*; Christoph Schweigert and Jürgen Fuchs, *D-Brane Conformal Field Theory and Bundles of Conformal Blocks*; Angel M. Uranga, *From Quiver Diagrams to Particle Physics.* **Mini-Symposium on Symplectic and Contact Geometry and Hamiltonian Dynamics:** Paul Biran, *From Sympletic Packing to Algebraic Geometry and Back*; Yuri Chekanov, *New Invariants of Legendrian Knots*; Hansjörg Geiges, *Contact Topology in Dimension Greater than Three*; Victor L. Ginzburg, *The Hamiltonian Seifert Conjecture: Examples and Open Problems*; Àngel Jorba and Jordi Villanueva, *The Fine Geometry of the Cantor Families of Invariant Tori in Hamiltonian Systems*; Mikhail B. Servryuk, *Sympletic and Contact Geometry and Hamiltonian Dynamics*; Vladimir Zakalyukin, *Simple Coisotropic Projections and Caustics.* **Mini-Symposium on Wavelet Applications in Signal Processing:** Eric Chassande-Mottin and Patrick Flandrin, *Reassigned Scalograms and Singularities*; Peter F. Craigmile, Donald B. Percival and Peter Guttorp, *The Impact of Wavelet Coefficient Correlations on Fractionally Differenced Process Estimation*; Emma J. McCoy, *Wavelet-Based Modelling of Persistent Periodicities*; Rudolf H. Riedi, Vinary J. Ribeiro, Matthew S. Course and Richard G. Baraniuk, *Network Traffic Modeling Using a Multifractal Wavelet Model*; Vasily Strela, *Denoising Vial Block Wiener Filtering in Wavelet Domain*; Andrew T. Walden, *Wavelet Analysis of Discrete Time Series.*

Peter Hajnal (Szeged)

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JON H. DAVIS, **Methods of Applied Mathematics with a MATLAB Overview**, XIV+721 pages, Birkhäuser, Basel – Boston – Berlin, 2004.

The subtitle of this textbook could be "Applications of Fourier analysis". It is based on the author's courses on the methods of applied mathematics for engineer, mathematics students and students of applied sciences. The treatment covers Fourier series, Laplace, Fourier and discrete transformation theories and their applications to ordinary and partial differential equations, mainly to boundary value problems, which appear in different applied areas. In addition to the topics covered by the authors' courses, the book contains an advanced chapter on discrete transforms and a chapter on waveform analysis and wavelets.

The theories are treated and also illustrated by a great number of examples and applications (e.g. in mathematical physics, control, and communications). Problems are stated to solve by the students from the very elementary to advanced levels, in which both the computational aspects and also the difficulties of the formal methods are emphasized. A modern and application-based treatment of these topics cannot be done without computing tools. The software MATLAB is used to illustrate the topics, solve and visualize the problems investigated. As the program codes are inserted in the text, the reader can learn the theory and its computer realization together. Unfortunately, the programs are not attached on a CD-ROM. To help the first steps in MATLAB a short introduction can be found in the Appendix.

The book is sectioned into nine main and three additional chapters. After a short introduction, the second chapter contains an introduction to the theory and simple applications of Fourier series, which is followed by more applications in Chapter 3 such as methods of elementary boundary value problems, for example 1D diffusion, wave equation, separation of variables. Then, Chapter 4 contains the Sturm-Liouville theory and higher dimensional nonrectangular boundary value problems. It also includes the elements of finite differences and numerical methods of solving BVP's. Since functions of complex variables are essentially involved in the following chapters, Chapter 5 contains complex analysis at the required level. Chapters 6 – 9 contains the properties of Laplace and Fourier transformations and their discrete versions. They are applied to differential and difference equations and then to solve applied problems in physics, engineering, etc. Finally, in Chapter 9, the reader can find some advanced topics such as local waveform analysis, wavelet applications. The appendix contains a linear algebra overview, an introduction to MATLAB and tables of transforms.

I can recommend the book to graduate students in mathematics, physics and also to students in applied sciences and engineering for their study of applications of Fourier analysis and differential equations.

János Karsai (Szeged)

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David Mumford, Caroline Series and David Wright, **Indra's Pearls: The Vision of Felix Klein**, XX+396 pages, Cambridge University Press, New York, 2002.

The book's main topic is the study of patterns that are simultaneously symmetrical under two Möbius maps. These patterns were discovered by Felix Klein in the nineteenth century and first described in the works of Klein and Fricke. Of course, all their illustrations were made by hand. Some of the sets were so intricate that —as Klein sadly confessed— "the imagination seems to fail utterly when we try to form a mental image of the result". This book is a reexamination of some of Klein's ideas with the aid of high speed computers and computer graphics. It can be read on many levels. One can browse through the numerous beautiful and fascinating pictures and marvel at them. Or one can study the mathematics involved. Finally, one can sit down in front of a computer and try to recreate the images by implementing the programs that are presented in pseudo-code. Readers with widely different backgrounds will find something enjoyable in this unique book.

The idea of Indra's net is found in an ancient Buddhist text and is described in the introduction as follows: "...the pearls in the net reflect each other, the reflections themselves containing not merely the other pearls but also the reflections of other pearls. In fact the entire universe is to be found not only in each pearl, but also in each reflection of each pearl, and so *ad infinitum*". Today we would use the word *self-similarity*, and this is one of the key concepts in the book.

Here is the table of contents: 1. The language of symmetry; 2. A delightful fiction; 3. Double spirals and Möbius maps; 4. The Schottky dance; 5. Fractal dust and infinite words; 6. Indra's necklace; 7. The glowing gasket; 8. Playing with parameters; 9. Accidents will happen; 10. Between the cracks; 11. Crossing boundaries; 12. Epilogue; Index; Road map of two-generator groups.

László I. Szabó (Szeged)

Alexandre V. Borovik, I. M. Gelfand and Neil White, **Coxeter Matroids** (Progress in Mathematics, 216), X+264 pages, Birkhäuser, Basel – Boston – Berlin, 2003.

Among the over fifty different equivalent formulations of a matroid, especially interesting are those which show the connection with combinatorial optimization. Edmonds' classical theorem characterizing matroids in terms of the greedy algorithm can be reformulated by Gale's maximality principle. This says that for every linear order $\leq$ on a finite set S there is a unique element in the set $\mathcal{B}$ of k-element subsets of S which is maximal with respect to the Gale order induced by $\leq$ if and only if $\mathcal{B}$ forms the set of bases of a matroid. Here the Gale order is a partial order defined as follows. Let $A = \{a_1, a_2, \ldots, a_k\}$ and $B = \{b_1, b_2, \ldots, b_k\}$ be elements of $\mathcal{B}$ such that $a_1 < a_2 < \cdots < a_k$ and $b_1 < b_2 < \cdots < b_k$ in the linear order above. Then in the Gale order, $A \leq_G B$ if and only if $a_1 \leq b_1$, $a_2 \leq b_2$, $\ldots$, $a_k \leq b_k$.

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This principle is a starting point for generalizing the notion of a matroid to Coxeter matroid. Why Coxeter? The name of the outstanding geometer of the 20th century, to the memory of whom the book under review is dedicated, enters the theory through the Coxeter groups. The notion of a Coxeter group is the mathematical abstraction of the kaleidescope. It is determined by a closed system of mirrors; closedness is meant so that the mirror image of a mirror in another mirror is a mirror that also belongs to the system. By the usual formal definition, a Coxeter group W is given by generators and relations as

$$W = \langle\, r_i \,|\, (r_i r_j)^{m_{ij}} = 1 \text{ for all } i, j \in I \,\rangle,$$

with some index set I, where $m_{ii} = 1$ and $m_{ij} \in \mathbb{Z} \cup \{\infty\}$ with $m_{ij} \geq 2$ if $i \neq j$. Not every Coxeter group, determined by the distinguished set of — abstract — involutory generators in this way, can be represented as a group of reflections in the sense above; but for a finite group such a representation always exists.

The simplest example of a finite Coxeter group is A_{n-1} ($n \geq 2$), which is essentially the same as the symmetric group S_n of degree n, the group of all permutations on the set $\{1, 2, \ldots, n\}$. The set of generators consists of the $n-1$ adjacent transpositions $r_i = (i, i+1)$. Take a subgroup P generated by some subset of these generators (for a general Coxeter group, such a subgroup is called a standard parabolic subgroup). In particular, we choose the subgroup obtained by omitting just one transposition from the generator system, that is when our generators are $r_1, \ldots, r_{k-1}, r_{k+1}, \ldots, r_{n-1}$, for some $k = 1, \ldots, n-1$. This subgroup is isomorphic to $S_k \times S_{n-k}$, the group that permutes the subsets $\{1, 2, \ldots, k\}$ and $\{k+1, k+2, \ldots, n\}$ in themselves. Thus, in the factor set $S_n/(S_k \times S_{n-k})$ there are altogether $n!/k!(n-k)!$ cosets and, for $u \in S_n/(S_k \times S_{n-k})$, the correspondence $u \mapsto u(\{1, 2, \ldots, k\})$ is a bijection from $S_n/(S_k \times S_{n-k})$ to the set of all k-element subsets of $\{1, 2, \ldots, n\}$. Moreover, this correspondence determines an order on $S_n/(S_k \times S_{n-k})$ corresponding to the Gale order on the set of k-element subsets.

Now the generalization to Coxeter matroids goes essentially in three steps. First, one replaces S_n by an arbitrary finite Coxeter group W. Then, from the set of the standard generators of W one may omit more than just one element. Thus, with an arbitrary subset $J \subseteq I$ one may choose the standard parabolic subgroup W_J of W generated by the set $\{r_i \,|\, i \in J\}$. Finally, instead of the Gale order — that we found to be present in our example $S_n/(S_k \times S_{n-k})$ above — one uses the "w-shifted Bruhat order".

As for this latter notion, first we note that here w stands for an element of W and it is only responsible for a given reordering. In particular, returning again to the Gale order we started from, obviously every linear order on the set S is determined by a permutation w that happens to be in $W = S_n$. Thus we put $A \leq^w B$ if and only if $w^{-1}A \leq^w w^{-1}B$, and the maximality principle cited above is reformulated in the following Theorem:

$\mathcal{B}$ is a matroid if and only if for every $w \in S_n$ the collection $\mathcal{B}$ contains a unique member $A \in \mathcal{B}$ maximal in $\mathcal{B}$ with respect to $\leq^w$.

In general, in a Coxeter group W the Bruhat order partially orders the underlying set of W. To compare elements $u, v \in W$, let both be given by the shortest possible word in the alphabet of the generators r_i given above. Then $u \leq v$ with respect to the Bruhat order if and only if the word for u can be obtained by deleting some generators from the word

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for v. Similarly as before, the w-shifted Bruhat order $u \leq^w v$ is defined by the relation $w^{-1}u \leq w^{-1}v$. Finally, both the Bruhat order and its shifted version induces an order on the factor set W/W_J of the cosets of W_J.

Now we are ready to define the Coxeter matroids, as follows. Let W be a finite Coxeter group, P a standard parabolic subgroup in W and $\leq$ the induced Bruhat order on the factor set $W^P = W/P$. Let $\leq^w$ denote the w-shifted order, $A \leq^w B$ if and only if $w^{-1}A \leq^w w^{-1}B$. Let $\mathcal{M}$ be a subset of W^P. We say that the set $\mathcal{M} \subseteq W^P$ is a *Coxeter matroid* if it satisfies the following Maximality Property: for any $w \in W$, there is a unique $A \in \mathcal{M}$ such that, for all $B \in \mathcal{M}$, $B \leq^w A$.

In accordance with that the generalization starts from ordinary matroids, in Chapter 1, after some preparatory sections dealing with them, one finds their reinterpretation in terms of the symmetric group S_n, the simplest example of a Coxeter group. The class of Coxeter matroids that are conceptually closest to ordinary matroids are flag matroids; thus they are introduced and described in some detail in the same chapter. A flag matroid is obtained from a Coxeter group $A_{n-1} \cong S_n$ when the standard parabolic subgroup in the definition above is not necessarily maximal.

Both matroid theorists and lattice theorists know that there is a common part of their specialty; this is the field of geometric lattices, a special class of semimodular lattices (the key statement here is that the closed sets of a matroid form a geometric lattice). Chapter 2 is devoted to this connection and it is shown that every flag matroid may be represented in a semimodular lattice, and even slightly better, in a geometric lattice.

What are the other Coxeter groups that appear in the theory? According to the classification of finite Coxeter groups due to Cartan and Coxeter, there are four infinite families: A_n, BC_n, D_n and $G_2(m)$, as well as six "sporadic" types: E_6, E_7, E_8, F_4, H_3 and H_4; all the other finite Coxeter groups are direct products of these types (this classification theorem is discussed in Chapter 7). We note that three of the infinite families consist of groups which can be represented geometrically as symmetry groups of certain regular polytopes.

Thus, A_n is the symmetry group of the regular n-simplex, and BC_n is the symmetry group of the n-cube, or dually, of the cross-polytope (the generalization of the regular octahedron). Taking BC_n and using one of its maximal parabolic subgroups for the construction one obtains a Coxeter matroid what is called a "symplectic matroid". This topic is developed in Chapter 3; here, in particular, it is shown that representable symplectic matroids arise from totally isotropic subspaces of a symplectic space, in a similar way as representable ordinary matroids arise from a subspace of a vector space. A further type, namely, symplectic flag matroids are also described here; these are analogues of the "ordinary" flag matroids.

At the end of the same chapter Coxeter matroids provided by the groups of type D_n are also introduced. Such a group is a subgroup of index 2 in BC_n; the corresponding D_n-matroids are called orthogonal matroids. They form a subclass of symplectic matroids.

Chapter 4 deals with a particular class of symplectic matroids, called Lagrangian matroids. It is interesting that such structures appeared in the matroid literature, histor-

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ically preceding the Coxeter matroid concept, under the names of symmetric matroids, 2-matroids, metroids, or Δ-matroids.

Chapter 5 contains a summary of the background material on reflection groups and Coxeter groups necessary for understanding the rest of the book, on half a hundred pages. About one-third of the figures of the book appears here, making this chapter an enjoyable reading for those who feel important the visual thinking in mathematics.

In Chapter 6 the theory of Coxeter matroids is developed in full generality. The special cases of the preceding chapters are also reviewed in this broader context. Also, the Gelfand-Serganova theorem, that occurred already in connection with the special cases is discussed here. This theorem, which is regarded by the authors as the cornerstone of the theory, provides a geometric characterization of Coxeter matroids. (We note that I. M. Gelfand and V. V. Serganova are just the initiators of the theory since they introduced the Coxeter matroid concept — originally under the name WP-matroids —, answering this way the question of J. P. S. Kung, the essence of which is as follows: can matroids be defined in terms of the symmetric groups? If so, what do the corresponding Coxeter group analogues look like?)

Let P be a standard parabolic subgroup of a finite Coxeter group W as above. Consider the usual geometric realization of W as a reflection group. Then there is a suitable Euclidean space $\mathbb{E}$ on which it acts as a group of orthogonal transformations, moreover, this action is essential, i.e. there is no non-zero fixed point. Let $\delta \in \mathbb{E}$ be a point stabilized by P. With any subset $\mathcal{M} \subseteq W/P$ we associate a polytope $\Delta_{\mathcal{M}}$ defined as the convex hull $\mathrm{conv}\{\overline{w}\delta \,|\, \overline{w} \in \mathcal{M}\}$.

On the other hand, let Δ be a convex polytope. For every edge e of Δ take the hyperplane that perpendicularly bisects e. Let $W(\Delta)$ be the group generated by the reflections in all such hyperplanes. We say that Δ is a *Coxeter matroid polytope* if the group $W(\Delta)$ is finite. Now the Gelfand-Serganova theorem states that for a finite Coxeter group W, a subset $\mathcal{M} \subseteq W/P$ is a Coxeter matroid if and only if $\Delta_{\mathcal{M}}$ is a Coxeter matroid polytope.

We have two remarks here. First, specialists of ordinary matroid theory may realize that the basis polytope of a matroid is just a special case of Coxeter matroid polytopes (other special cases are discussed in the same chapter as well). On the other hand, those who have some experience concerning the works of H. S. M. Coxeter immediately notice that for $\mathcal{M} = W/P$, the Coxeter matroid polytopes are essentially the same as the regular and semiregular (more generally, *uniform*; but this term is ignored here) polytopes. By the way, it is a bit strange for the reviewer to see that in a book like this (see the Dedication!) there is not a single reference to some original work of Coxeter; not even to any one from the series of his papers in which the polytopes in question are constructed and studied in detail.

The final Chapter 7 deals with a problem the classical version of which is important in matroid theory as well: this is the representability problem. Here the tools applied in Chapter 2 are developed further and it is shown that in a more general approach the theory of buildings provides a natural context to tackle this problem. Necessarily, the first part of the chapter contains a brief summary of the basics of buildings, an interesting

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class of structures that was originally introduced by Tits with the purpose of a better understanding of certain classes of groups.

Every chapter is completed with exercises and some of them with a list of problems as well. By the intention of the authors, this book was written both for graduate students and research mathematicians in combinatorics or in algebra. However, anyone, with a suitable degree of maturity in mathematics, may enjoy the presentation — written in vivid style — of the results in a new area of research, not far from being *in statu nascendi* (a great majority of papers on the relevant results cited in the book is not older than a decade or so). Future shows how the interlocking of two extended fields of research, the theory of Coxeter groups and of classical matroids, developed independently, will generate a new, colourful, expectedly exciting area of mathematics.

Gábor Gévay (Szeged)

Bernd S. W. Schröder, **Ordered sets, An introduction**, XVIII+391 pages, Birkhäuser, Boston, 2003.

The notion of order occurs in all areas of mathematics. This explains why it is hard to write a comprehensive introductory book on ordered sets. As far as the reviewer knows, up to now this is the first book that attempts to give an introduction to the theory of orders. The topics considered by the author form an important part of the theory of ordered sets. It is fair to say that the book reviewed is an introduction to some parts of the theory of ordered sets.

The main topics considered in the book are chain decomposition, fixed points, reconstruction, dismantlability, completion of posets, dimension, arithmetic of ordered sets, enumeration and constraint satisfaction.

To make the readers interested through the reading, the author chooses some unsolved problems, such as characterization of the fixed point property and reconstruction of ordered sets and shows off how the basic definitions and theorems work in proving special cases of these problems. The author sticks to this theme through the whole text. At certain points the reviewer found this method of presentation tiring. In the beginning of each section the author gives an outline of what the intended material covered in the section is. In a few cases his explanations run away and give obscure information. Exercises at the end of each chapter may help interested students to acquire the material.

The text contains a few examples of interaction of the theory of ordered sets and other disciplines of mathematics. E. g., Lefshetz's fixed point theorem is used to obtain a fixed point theorem of ordered sets.

The organization of topics in the book is clear and concise. The book is a useful tool for introductory courses in the theory of ordered sets. It may also inspire other authors in the field of ordered sets to prepare their introductory books of important topics not included in the book reviewed, such as the relationship between finite topologies and orders in connection with algebraic topology, order varieties, monotone operations and

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clones, and compatible order relations in algebras.

László Zádori (Szeged)

HUBERT KIECHLE, **Theory of K-loops** (Lecture Notes in Mathematics, 1778), X+186 pages, Springer-Verlag, Berlin, 2002.

The theory of loops and quasigroups was born in the 1930's; at that time the motivations were mostly geometric. Nowadays, this area gains more and more importance in different branches of mathematics: foundations of geometry, differential geometry, general algebra, group theory, combinatorics and mathematical physics. In particular, the class of K-loops is located in the crossing of these fields, despite the fact that the identities defining this class look rather harmless. The approach of this book is based on the foundations of geometry and on group theory but the Appendix contains a short history of K-loops, where many references are given and the other approaches are also mentioned.

The book is self-contained, the only prerequisite is basic algebra, in particular, group theory. Chapters 1–6 unfold the theory in a coherent way, and could be used as a text. With very few exceptions, complete proofs are given. Chapters 7–11 are more like research notes, and only partially suitable for the classroom. Still, proofs are concise but complete. Many examples are given in Chapters 9,11,12; they constitute a special value of the book.

This well written book can serve as a reference and a source of constructions for researchers of the theory of non-associative structures. Also, thanks to the didactic presentation, I can also recommend it for motivated graduate students and university teachers.

Gábor P. Nagy (Szeged)

PAUL F. X. MÜLLER, **Isomorphisms between H^1 spaces**, XIV+453 pages, Birkhäuser Verlag, Basel – Boston – Berlin, 2005.

The theory of Hardy spaces is firmly rooted in the fields of complex analysis, trigonometric series, and probability. The classical theory of Hardy spaces is concerned with the boundary behavior of analytic functions on the unit disk and with estimates for the Fourier coefficients of the limit function on the unit circle. Its history began with the seminal papers by G.H. Hardy (1915), F. and M. Riesz (1916), A.N. Kolmogorov (1925), G.H. Hardy and J.E. Littlewood (1930) and R.E.A.C. Paley (1933). In the 1920's, probability was also confronted with analogous problems of the laws of large numbers. In this way emerged martingales, martingale convergence theorems, and J.L. Doob's maximal function.

By 1977 there existed two highly developed classes of H^1 spaces. These were the atomic H^1 spaces which are linked to analytic functions via C. Fefferman's duality theorem, and the martingale H^1 spaces consisting of martingales for which Doob's maximal function is integrable. A remarkably simple object, called dyadic H^1, appears in both classes.

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The book under review presents a thorough and self-contained account of H^1 and its known isomorphic invariants. Throughout the book, a special attention is given to the combinatorial methods developed in the field. In particular, J. Bourgain's proof of the dimension conjecture, L. Carleson's biorthogonal system in H^1, T. Figiel's integral representation, W.B. Johnson's factorization of operators, B. Maurey's isomorphism, and P. Jones' proof of the uniform approximation property are treated in full details. To achieve a better understanding, an entire chapter is devoted to the study of combinatorics of colored dyadic intervals.

This book was written under the author's conviction that the Haar system introduced by A. Haar (1910) plays a decisive role in the whole theory, which is fully justified by its impact on wide ranging areas of analysis and probability. For example, the Haar system is clearly the simplest martingale difference sequence, yet by B. Maurey's theorem, every martingale is conveniently embedded in the partial sums of a Haar series. Another example is the following: Calderón–Zygmund operators can be expressed as paraproducts plus an absolutely convergent series of basic operators rearranging and splitting the Haar system. This is the content of T. Figiel's integral representation. Last but not least, the Haar system is a reliable instrument for separating core ideas from their surrounding technicalities.

Table of contents. *Preface.*

1. *The Haar System: Basic Facts and Classical Results.* 1.1. Bases in L^p, 1.2. Dyadic H^1, 1.3. Bounded square functions and large deviation inequalities, 1.4. Martingales and biorthogonal systems, 1.5. Basic operators.

2. *Projections, Isomorphisms and Interpolation.* 2.1. Complemented subspaces, 2.2. Pelczyński's decomposition method, 2.3. Interpolation of operators.

3. *Combinatorics of Colored Dyadic Intervals.* 3.1. The Carleson packing condition, 3.2. Orthogonal projections and colored intervals, 3.3. Rearrangement operators.

4. *Martingale H^1 Spaces.* 4.1. Maurey's isomorphism, 4.2. Isomorphic classification, 4.3. More on subsystems of the Haar system.

5. *Isomorphic Invariants for H^1.* 5.1. Complemented copies of Hilbert spaces, 5.2. Complemented copies of H^1_n, 5.3. The uniform approximation property of BMO.

6. *Atomic H^1 Spaces.* 6.1. Basic similarities between H^1 and H^1_{at}, 6.2. Carleson's biorthogonal system, 6.3. Spaces of homogeneous type, 6.4. Orthogonal projections in atomic H^1 spaces, 6.5. Martingale approximation in atomic H^1 spaces.

Bibliography comprising 217 items, *List of Symbols*, and *Index*.

Each chapter ends with Notes which provide a wealth of information on the history of the subject and contain a number of related results and methods.

The necessary background is also contained in the book. This includes a detailed discussion of the Haar system, together with the operators that can be built from it, such as averaging projections, rearrangement operators, paraproducts, Calderón–Zygmund integrals. Complete proofs are given for the classical martingale inequalities of C. Fefferman, Burkholder, and Khinchine–Kahane, as well as for large deviation inequalities. Complex interpolation, analytic families of operators, and the Calderón product of Banach lattices are also treated in the context of H^p spaces.

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To sum up, this book contains all the basic theorems and methods of the fast-growing topic indicated in the title. Its style is concise, but always clear. We warmly recommend it to Ph.D. students and researchers dealing with Hardy spaces from the point of view of Analysis or Probability.

Ferenc Móricz (Szeged)

Israel Gohberg, Seymour Goldberg and Marinus A. Kaashoek, **Basic Classes of Linear Operators**, XVII+423 pages, Birkhäuser, Basel – Boston – Berlin, 2003.

From the preface: "The present book is an expanded and enriched version of the text *Basic Operator Theory*, written by the first two authors more than twenty years ago. Since then the three of us have used the basic operator theory text in various courses. This experience motivated us to update and improve the old text by including a wider variety of basic classes of operators and their applications. The present book has also been written in such a way that it can serve as an introduction to our previous books *Classes of Linear Operators*, Volumes I and II. We view the three books as a unit." The authors aim to give a firm foundation in operator theory. The subject material can be considered as a natural continuation of the undergraduate courses in linear algebra and analysis. As the authors write: "a great deal of attention is paid to the interplay between abstract methods and concrete problems. Motivation of the theory and its systematic applications are characteristic of all chapters." Furthermore: "One of our aims is to reestablish a theory of integral equations as a significant part of operator theory." The chapters are: I. Hilbert Spaces, II. Bounded Linear Operators on Hilbert Spaces, III. Laurent and Toeplitz Operators on Hilbert Spaces, IV. Spectral Theory of Compact Self Adjoint Operators, V. Spectral Theory of Integral Operators, VI. Unbounded Operators on Hilbert Space, VII. Oscillations of an Elastic String, VIII. Operational Calculus with Applications, IX. Solving Linear Equations by Iterative Methods, X. Further Developments of the Spectral Theorem, XI. Banach Spaces, XII. Linear Operators on a Banach Space, XIII. Compact Operators on a Banach Space, XIV. Poincaré Operators: Determinant and Trace, XV. Fredholm Operators, XVI. Toeplitz and Singular Integral Operators, XVII. Non Linear Operators. All chapters end with a long list of exercises.

This book, written by leading experts of the field, can be warmly recommended to undergraduate and graduate students of mathematics, engineering, and other technical sciences.

L. Kérchy (Szeged)

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CARLOS S. KUBRUSLY, **Hilbert Space Operators, A Problem Solving Approach**, XIII+149 pages, Birkhäuser, Basel – Boston – Berlin, 2003.

It is common sense that the best and most efficient way of learning mathematics is problem solving. This volume gives a problem solving approach to the theory of Hilbert space operators. It consists of the following chapters: 1. Invariant Subspaces, 2. Hilbert Space Operators, 3. Convergence and Stability, 4. Reducing Subspaces, 5. Shifts, 6. Decompositions, 7. Hyponormal Operators, 8. Spectral Properties, 9. Paranormal Operators, 10. Proper Contractions, 11. Quasireducible Operators, 12. The Lomonosov Theorem. Each chapter starts with giving the basic definitions and fixing the notation. Then, after providing motivation, the problems are formulated. Finally, the chapters end with complete solutions to the problems. Only knowledge of elementary functional analysis is needed as prerequisite.

This book is primarily aimed at graduate students, both in pure and applied mathematics.

L. Kérchy (Szeged)

VLADIMIR MÜLLER, **Spectral Theory of Linear Operators and Spectral Systems in Banach Algebras** (Operator Theory: Advances and Applications, Vol. 139), X+381 pages, Birkhäuser, Basel – Boston – Berlin, 2003.

Spectra of elements in Banach algebras or of operators acting on Banach spaces bear a lot of information and play important role in many areas of mathematics and physics. We can get such a notion by specifying a subset R of *regular elements* of the Banach algebra $\mathcal{A}$, and then defining the corresponding spectrum $\sigma_R(a)$ of an element $a \in \mathcal{A}$ as the set of complex numbers λ with the property that $a - \lambda$ does not belong to R. In this volume an axiomatization is given for that process, and on that basis a comprehensive survey of various spectra is carried out. The book consists of the following chapters: I. Banach Algebras, II. Operators, III. Essential Spectrum, IV. Taylor Spectrum, V. Orbits and Capacity. Each chapter ends with extensive historical comments and further aspects of the theory.

This monograph can be warmly recommended to graduate students who would like to learn about spectral theory. It can also serve as a reference book for experts on this field.

L. Kérchy (Szeged)

VLADIMIR V. PELLER, **Hankel Operators and Their Applications** (Springer Monographs in Mathematics), XV+784 pages, Springer, New York, 2003.

A transformation on the sequence space ℓ^2 is called a Hankel operator if its matrix entries depend only on the sum of the indices. The starting point of the modern theory

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of Hankel operators was the classical theorem of Z. Nehari from 1957, identifying the bounded Hankel operators with those associated with functions in the space L^∞. The Hankel operator corresponding to the symbol $\varphi \in L^\infty$ can be realized as the mapping $H_\varphi: H^2 \to L^2 \ominus H^2,\ f \mapsto (I - P_+)\varphi f$, where P_+ stands for the orthogonal projection onto the Hardy subspace H^2 of L^2. These operators turned out to be useful tools in many different areas of mathematics, including approximation theory, prediction theory and control theory. This book gives a systematic account of the tremendous development, the theory underwent in the last half century. The following topics are covered in the fifteen chapters: 1. An Introduction to Hankel Operators, 2. Vectorial Hankel Operators, 3. Toeplitz Operators, 4. Singular Values of Hankel Operators, 5. Parametrization of Solutions of the Nehari Problem, 6. Hankel Operators and Schatten–von Neumann Classes, 7. Best Approximation by Analytic and Meromorphic Functions, 8. An Introduction to Gaussian Spaces, 9. Regularity Conditions for Stationary Processes, 10. Spectral Properties of Hankel Operators, 11. Hankel Operators in Control Theory, 12. The Inverse Spectral Problem for Self-Adjoint Hankel Operators, 13. Wiener–Hopf Factorizations and the Recovery Problem, 14. Analytic Approximation of Matrix Functions, 15. Hankel Operators and Similarity to a Contraction. Each chapter ends with Concluding Remarks, providing historical background and further references. Necessary information on Hilbert space operators and function spaces are collected in two Appendices.

This volume will certainly serve as a standard reference on Hankel operators. It can be warmly recommended to graduate students as well as experts in analysis and operator theory.

L. Kérchy (Szeged)

Jacques Tits and Richard M. Weiss, **Moufang polygons** (Springer Monographs in Mathematics), X+535 pages, Springer-Verlag, Berlin, 2002.

Since 1974, in a series of papers and lecture notes Jacques Tits introduced the notion of buildings. Buildings are very useful combinatorial objects with classical geometric motivation for dealing with algebraic and abstract simple groups. Generalized polygons correspond to a special class of buildings, the irreducible spherical buildings of rank two. Generalized polygons are too numerous to classify, the Moufang condition ensures that the automorphism group of the polygon acts highly transitively. In this excellently written book, the authors give a full classification for Moufang polygons. Roughly speaking, one may say that every Moufang polygon is classical, algebraic or mixed.

The book is organized as follows. In Part I, the authors introduce Moufang n-gons and show that a Moufang n-gon is uniquely defined by an ordered sequence of "root groups" together with certain commutator relations. In Part II, they describe nine families of examples. Part III contains the classification itself. In Part IV, they examine the automorphism group of a Moufang polygon in order to deal with the problem of isomorphism. In Part V, they turn to buildings and use the knowledge of Moufang polygons to give a simplified classification of thick irreducible spherical buildings of rank

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at least three. Moreover, they give a brief overview of Moufang polygons as they occur as classical, algebraic and mixed buildings. In the Appendix, the connection between spherical buildings and algebraic groups is recalled and used to describe an existence proof for the exceptional Moufang polygons.

The book is self-contained, beside graduate algebra, no prerequisites are needed. The areas it covers belong to the most recent ones in the mathematical research: classical geometries and their groups, algebraic groups, field extensions and combinatorics. The notions and proofs are certainly deep but due to the elegant presentation and the numerous remarks and explanations, the content of the book is accessible for motivated graduate students and researchers from every branch of mathematics. We recommend this book for everybody who is interested in the developments of the modern algebra, geometry or combinatorics.

Gábor P. Nagy (Szeged)

INGO WEGENER, **Complexity Theory, Exploring the Limits of Efficient Algorithms** (Translated from the German by Randall Pruim), XII+308 pages, Springer-Verlag, Berlin, 2005.

The present book is translated from the German "Komplexitätstheorie - Grenzen der Effizienz von Algorithmen" (Springer-Verlag 2003), by Randall Pruim. For those who know the German original we quote the translator "Most of the revisions to the book are quite minor. Some bibliography items have been added or updated; a number of German sources have been deleted. Occasionally I have added or rearranged a paragraph, or included some additional detail, but for the most part I have followed the original quite closely. Where I found errors in the original, I have tried to fix them; I hope I have corrected more than I have introduced."

Complexity theory is an extremely important and vivid field on the border of mathematics and computer science. It is an important part of the syllabus of math and CS majors. The author describes the motivation of his undertaken project as follows: "At least since the development of the theory of NP-completeness, complexity theory has become a central area of instruction and research within computer science. The $NP \neq P$-problem represents one of the great intellectual challenges of the present. In contrast to other areas within computer science, where it is often suggested that nearly all problems are solvable with the aid of computers, the goals of complexity theory include showing what computers cannot do. Delineating the boundary between problems that can be efficiently solved and those that can only be solved with an unreasonable amount of resources is a practically relevant question, but so is the structural question of what determines the complexity or 'complicatedness' of problems."

"The development of complexity theory is presented in this book as essentially a reaction to algorithmic development. For this reason, the investigation of practically important optimization problems plays a predominant role. From this algorithmic perspective, reduction concepts can be thought of as methods to solve problems with the

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help of algorithms for other problems. From this it follows conversely that we can derive the difficulty of a problem from the difficulty of other problems."

The contents well describe the wide range of topics covered in the book: 1. Introduction, 2. Algorithmic Problems & Their complexity, 3. Fundamental Complexity Classes, 4. Reductions – Algorithmic Relationship Between Problems, 5. The Theory of NP-Completeness, 6. NP-complete and NP-equivalent Problems, 7. The Complexity Analysis of Problems, 8. The Complexity of Approximation Problems – Classical Results, 9. The Complexity of Black Box Problems, 10. Additional Complexity Classes, 11. Interactive Proofs, 12. The PCP Theorem and the Complexity of Approximation Problems, 13. Further Topics From Classical Complexity Theory, 14. The Complexity of Non-uniform Problems, 15. Communication Complexity, 16. The Complexity of Boolean Functions, Final Comments, A. Appendix, References, Index.

The classical notions and results are combined with recent achievements of the field, machine based approach is mixed with combinatorial, algebraic and probabilistic approaches.

Finally we quote the author about the advantage of a translator. "It is possible to write a research monograph in a non-native language. In fact, I have done this before. But a textbook with a pictorial language needs a native speaker as translator. Moreover, the translator should have a good feeling for the formulations and a background to understand and even to shape and direct the text." Randall Pruim is an experienced translator who is very skilled. "This book is the second translation project I have undertaken for Springer. My goal each time has been to produce a text that will serve its new audience as well as the original book served its audience. Thus I have tried to mimic as far as possible the style and 'flavor' of the original text while making the necessary adaptations. At the same time, a translation affords an opportunity to make some improvements, which I have done in consultation with the original author. And so, in some sense, the result is a translation of a second edition that was never written."

Ingo Wegener certainly created an appealing, well-written book that is a definite choice for the specialists and lecturers when an undergraduate or graduate student asks for guidance into this challenging new field of mathematics.

Péter Hajnal (Szeged)

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